APPLICATIONS OF GRAPH THEORY

(A project report submitted for the degree of b.sc honours in mathematics under the University of Burdwan)

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ABSTRACT

Here in this project work, we have discussed some topics related to graph theory and then we have described its applications. We have solved and shown some problems by representing it as a graph. Different types of graphs and their examples and drawings have been presented here. Among vast applications of graph theory, some renowned problems and their uses have been chosen to explain in this project so that the importance of graph theory in our real life can be illustrated here beautifully.

CONTENT

SECTION – 1

INTRODUCTION

Graph theory is an extensive and popular branch of mathematics evolved in the process of finding solutions of different puzzles, games, popular problems and conjectures. As we see any physical problems involving discrete objects and relationships among them by a graph. Then we can proceed mathematically to find their solutions. Graph theory is applied in other subjects like Physics, Chemistry, Biological sciences, Sociology, Geography etc. It is being used in Engineering and Computer Science also. So for its vast of applications, the interest in graph is increasing day by day. In fact, high speed digital computer is one of the reasons for the recent growth of interest in graph theory. Almost any problem related to our real life involving discrete objects and a relation among the can be converted in a problem of graph theory. So graph theory is an important chapter in mathematics and also in our life.

1.1 HISTORY: Konigsberg's Bridge Problem :

There is an interesting story behind the development of the Graph Theory. There was a puzzle in the people of the city of Konigsberg in Prussia. The city was extended along both the sides of the Pregel river together with two large islands. The islands and two main lands of the city were connected by seven bridges.

Every Sunday afternoon, to walk across the entire city the people of Konigsberg tried to return to the starting point after starting their walk at anyone of the lands by crossing each of the seven bridges exactly once but they failed. No one was able to find such a walk. In 1736, Leonhard Euler first proved that such a walk did not exist. It was the first theorem of the graph theory as well as the theory of networks.

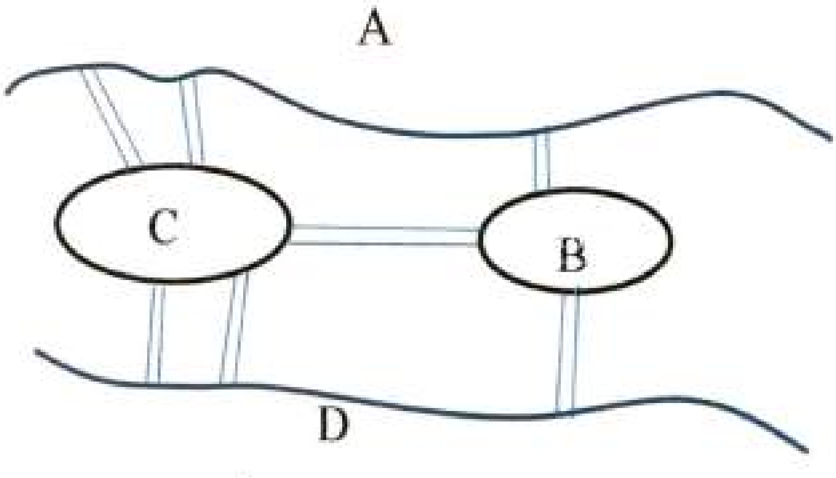


fig : 1

The Konigsberg's seven bridges problem is shown in fig no 1. we denote the main lands by A and D and Islands by B andC. The people of Konigsberg tried to start from anyone of the lands A, B, C, D for walking over each of the seven bridges exactly once to return to the starting point; but they were not successful and no one could explain it. This problem was adopted by Euler as a problem of grapy theory. Euler represented it as a graph of order four and size seven, where the vertices of the graph were represented by the lands A, B, C, D and the edges e_1, e_2 ,….., e_7 were represented by the seven bridges. The pictorial representation of the graph is given below:

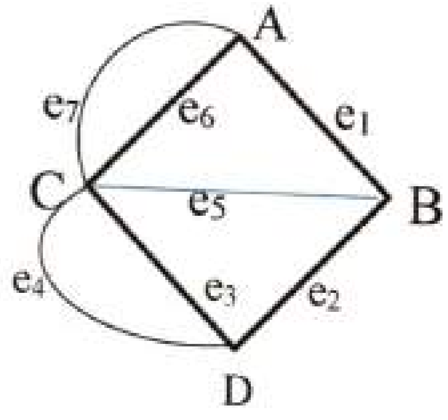


fig : 2

1.2 Prelimineries:

There are some definitions and properties of graphs for better understanding the applications of graphs.

1.2.1 DEFINITION OF A GRAPH

A graph G = (V, E) consists of

1) A non – empty set V of objects called its vertices, denoted by v_i and

2) Another set E of objects called its edges denoted by e_k.

3) The component set E may be empty and if e_k belongs to E, then e_k is associated with a pair of distinct vertices of G is called end vertices of e_k

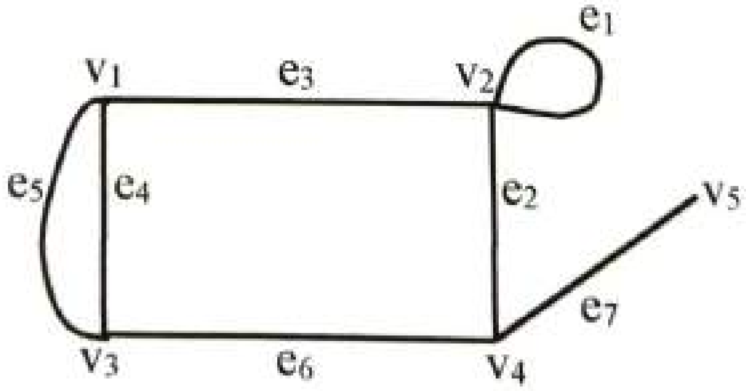


fig : 3

1.2.2 DIFFERENT TYPES OF GRAPHS

- **Pseudo graph and loops**

A Pseudo graph is like a graph but it may contain loops as well as parallel edges.

Example : Let $G = (V, E)$ where $V = (v_1, v_2, v_3, v_4, v_5)$ and $E = (e_1, e_2, \ldots\ldots\ldots e_7)$ as represented by

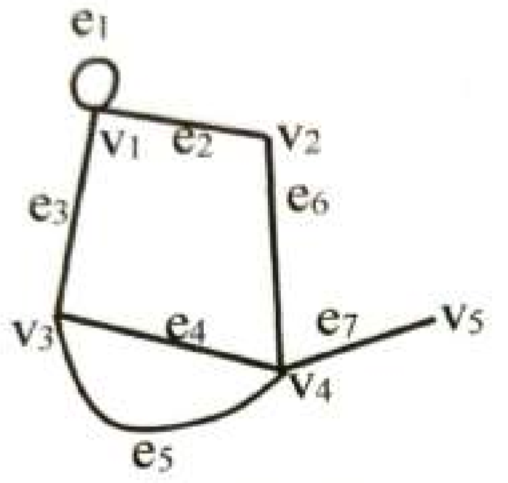


fig : 4

Here the graph $G = (V, E)$ as represented above is such a graph that the edge e_1 having the end vertices coincident at v_1. This type of edge is called a loop.

Also in the above graph edges e_4 and e_5 have the same and vertices, namely v_3 and v_4 edges are called parallel edges.

- **Null Graph**

In a graph G = (V, E), it is possible for the edge set E to be empty. Such a graph, without any edges, is called a Null graph.

In other words, every vertex in a null graph is a isolated vertex. Let G = (V,E,Ψ) be a graph with V= (v_1 ,v_2, v_3) and E = Φ. Then G is a null graph which is shown in the fig no. 5

fig : 5

- **Directed Graph**

A graph G = (V, E) is said to be a directed graph if its every edge $e_k = (v_i, v_j)$ is represented as a line segment from vertex v_i to v_j with an arrow directed from v_i to v_j .

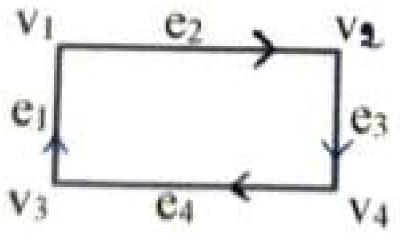


fig : 6

- **Regular Graph**

A graph G = (V, E) is called a regular graph it all its verities are of equal degrees.

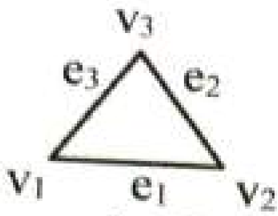


fig: 7

- **Simple Graph**

A graph G = (V, E) is called a simple graph if G has neither a loop nor parallel edges.

For example, any triangle or a polygonal graph is a simple graph.

- **Complete Graph**

A simple graph G is said to be a complete graph, it every pair of distinct vertices in G is joined by unique edge.

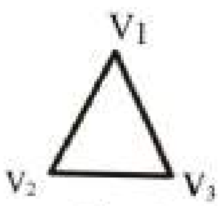


fig: 8

- **Cycle Graph**

A graph G is said to be cycle graph if it consists of a single cycle and a non-trivial closed path is called cycle.

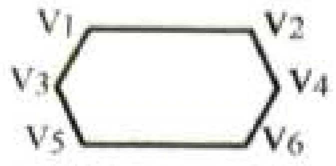


fig: 9

- **Bipartite Graph**

A bipartite graph is one whose vertices can be partitioned into two disjoint sets V_1 and V_2 , called bipartition sets in such a way that every edge joins a vertex inV_1 and a vertex in V_2. In particular there are no edges within V_1 nor within V_2.

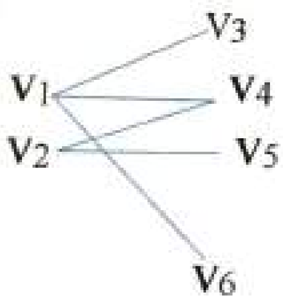


fig: 10

A complete Bipartite graph is a bipartite graph in which every vertex in V_1 is joined to every vertex in V_2.

➢ Subgraph

A graph g is a subgraph of another graph G if the vertex and the edge sets of g are respectively subsets of the vertex and edge sets of G.

For an Example,

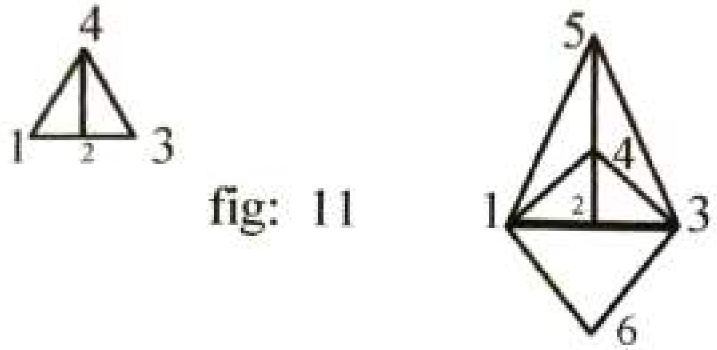


fig: 11

The first one is a subgraph of second one.

➢ Connected Graph

A graph G = (V, I) is said to be connected if there is at least one path between every pair of vertices in G.

If a graph G = (V , E) is not connected, then it is called disconnected.

➢ Weighted Graph

A weighted graph is a graph G (V, E) together with a function wiE $\rightarrow$[0, ∞). If e is an edge, the non-negative real number w(e) is called 5 weight of e. the weight of a subgraph of G is the sum of weights of the edges of the subgraph.

1.2.3 Isomorphisms of Graph

Two graphs $G = (V_1, E_1)$ and $G = (V_2, E_2)$ are said to be isomorphic if there is a one-to-one correspondence between elements of V_1 and those of V_2 and between element of E_1 and those of E_2 such that relation of incidence between vertices and edges remains unchanged.

Example : Suppose the edge $e \in E_1$ is incident with $v_i, v_i \in V_1$ in G_1 , then corresponding edge $e' \in E_2$ must be incident with $v_i', v_j' \in V_2$ where v_i' and v_j' are elements in V_2 corresponding to v_i and v_j respectively. Consider an example,

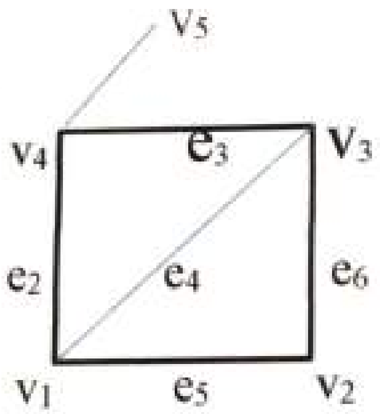


fig: 14

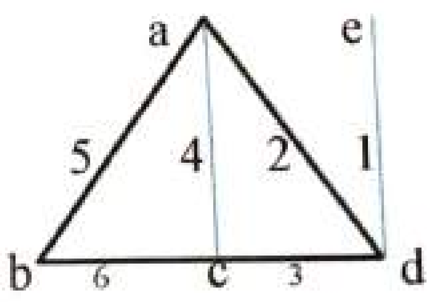


fig: 15

Consider the correspondence : $a \longleftrightarrow v_1$, $b \longleftrightarrow v_2$, $c \longleftrightarrow v_3$, $d \longleftrightarrow v_4$, $e \longleftrightarrow v_5$

1.2.4 PATHS AND CYCLE

- **Walk :**

A walk is defined as a finite alternating sequence of vertices and edges beginning and ending with vertices in a graph such that each edge is incident with vertices preceding and following it. In a walk, a vertex may appear more than one.

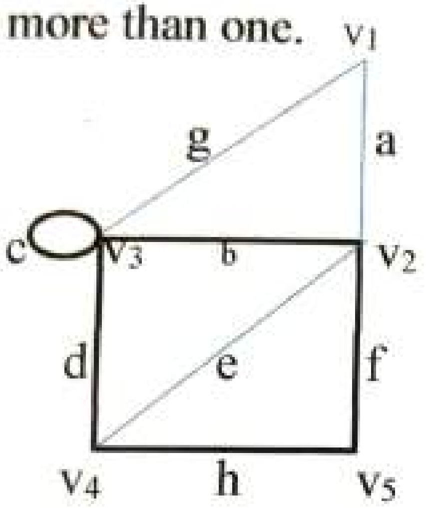


fig.16

Here $v_1 \xrightarrow{a} v_2 \xrightarrow{b} v_3 \xrightarrow{d} v_4 \xrightarrow{e} v_2 \xrightarrow{f} v_5$ forms a walk in G_1. Also the graph admits more than one walk.

- **Trail :** A walk having no repeated edges i.e all edges are distinct is called a trail. A trail is said to be closed, if its starting and terminal vertices coincide. The length of a trail is the total number of edges occurred in the trail. A closed trail is known as **Tour**.

In fig 16 $v_1 \xrightarrow{a} v_2 \xrightarrow{f} v_5$ is an example of trail and

$v_1 \xrightarrow{a} v_2 \xrightarrow{f} v_5 \xrightarrow{h} v_4 \xrightarrow{d} v_3 \xrightarrow{g} v_1$ is a tour v_1.

- **Path :**

A trail having non-repeated vertices, except possibly the starting and terminal vertices is called a path. Thus, a walk having no repeated vertices and edges, except the starting and end vertices will be a path.

A path with zero length is known as trivial path, otherwise it is non-trivial. A path is said to be closed, if its starting and terminal vertices coincide.

For example, in graph as given in fig 16, $v_1 \xrightarrow{a} v_2 \xrightarrow{b} v_3 \xrightarrow{d} v_4$

is a path but walk is described by $v_1 \xrightarrow{a} v_2 \xrightarrow{b} v_3 \xrightarrow{c} v_3 \xrightarrow{d} v_4 \xrightarrow{e} v_2 \xrightarrow{f} v_5$

is not a path.

➢ **Circuit** : A non-trivial closed trail is called a circuit. Since circuit is non-trivial closed trail, its length must be non-zero.

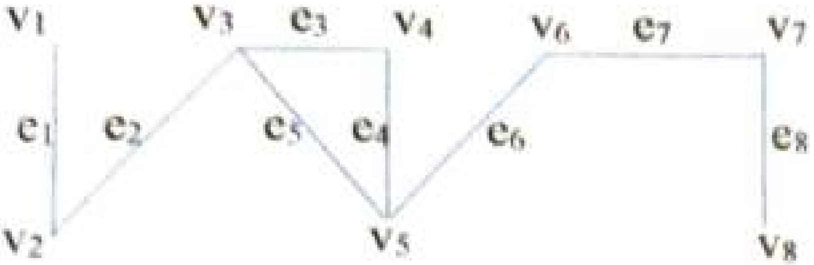


fig : 17

If fig 17, $(v_3 \xrightarrow{e_3} v_4 \xrightarrow{e_4} v_5 \xrightarrow{e_5} v_3)$ is a circuit. All the loops in a graph are circuit but the converse is not true.

➢ **Cycle :**

A non-trivial closed path is called a cycle. A cycle is said to be even if its length is even and it is called an odd cycle if its length is odd.

For example, in fig 17, $v_3 \xrightarrow{e_3} v_4 \xrightarrow{e_4} v_5$ is an odd cycle. Hence, every cycle is a circuit but the converse is not true.

- **Eulerian Trail :**

A closed trail containing all the edges of a graph G is called an Eulerian trail of the graph.

- **Eulerian Graph :**

A graph G is said to be Eulerian if it contains an Eulerian trail.

An Eulerian graph G is always connected except for isolated vertices of G that may exist. We shall always consider Euler's graph always devoid of isolated vertices and therefore connected.

However, there are connected graphs which are not Euler's graph.

Now we will discuss Euler's theorem.

- **Euler's Theorem :**

A connected graph G = (V, E) is an Euler's graph is if and only if all vertices of G are of even degrees.

Proof :- Suppose G = (V,E) is an Euler's graph. So G has an Euler's line is a closed path. In this path, every time we meet a vertex, v. We move through two edges in G incident with v, one entering v and other exerting v. This is also true for the terminal vertex. Hence each vertex of G has degree even.

Conversely, let all vertices of G be of even degrees, let v ∈ V. Let us construct a walk starting of v and going through edges of G such that no edge is traversed more than once.

We can continue tracing as far as possible. Since every vertex is of even degree, one can exist from every vertex entered upon. So tracing does not stop until we hit v again.

If this closed walk, say W includes all the edges of G, we have therefore finished, i.e G is an Euler's graph. If not, let us remove from G all the edges in W and obtain a subgraph W′ of G formed by removing edges. Since degree of the vertices in W′ is also even and $W \cap W' = \Phi$ (G is connected) taking v as a common vertex of W and W′, we can construct a new walk in W′ starting from v′ . By the same arguments as above this new walk shall terminate at v'.

Let us consider this walk with the previous one to compose a new walk starting and ending at V and of course having more edges.

This process is repeated until we obtain a close walk that includes all the edges of G. Hence G is an Euler's graph.

- **Solution of Konigsberg's Bridge problem :**

Solution of Konigsberg's Bridge Problem: Now if we look at the graph of the Konigsberg's bridges (fig 1), we Find that not all its vertices are of even degree. Hence, it is not an Euler's graph. Thus it is not possible to walk over each of the seven bridges exactly once and return to the starting point.

Euler has represented this situation by means of a graph given below.

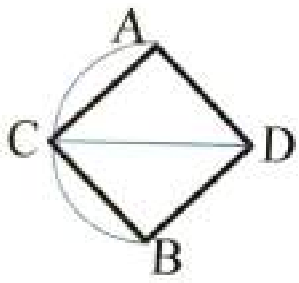


fig : 18

Hence it's not an Euler's graph and the answer to the problem is in negative.

- **Hamilton Graph :**

A Hamilton circuit (or cycle) in a connected graph G is defined as closed once except of course the starting vertex at which walk terminates. A graph G that contains Hamiltonian cycle is known as a Hamiltonian graph.

fig : 19

A Hamiltonian circuit for the graph in fig 19 is shown by bold lines.

- **Hamiltonian path:**

If one removes one edge from a Hamiltonian circuit, the result is called a Hamiltonian path.

So a Hamiltonian path traverses every vertex of a graph G, and hence a graph G that possesses a Hamiltonian circuit necessarily has a Hamiltonian path.

The length of a Hamiltonian path in a connected graph of n vertices is (n - 1)

1.2.5 <u>Tree</u>

➢ **Definition of a tree :**

A tree is a connected graph without any circuits. For example,

fig 20: trees with two, three and four vertices

➢ **Spanning Tree :**

A tree T is said to be a spanning tree of a connected graph G if T is a subgraph of G and if T contains all vertices of G.

Let T be a spanning tree of a connected graph G. Then every edge of T is known as a branch of T; an edge e of the connected graph G is said to be a chord of the spanning tree T of G, if e is not an edge of T. A chord is sometimes referred to as a link or tie.

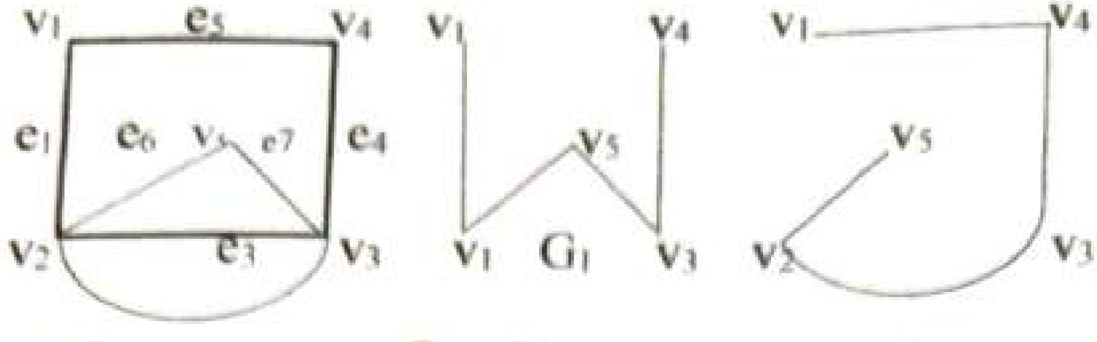


fig : 21

Here, G is connected graph of order 5 and G_1, G_2 are two district spanning trees of G, because G_1 and G_2 both spanning subgraphs of G. Every edge of G_i; is a branch of G_i (i = 1, 2). e_2 e_3, e_5 of G_1 are the chords of G and e_1, e_3, e_7 are the chords of G_2.

- **Rooted Tree :**

A tree is rooted if it comes with a specific vertex, called the root.

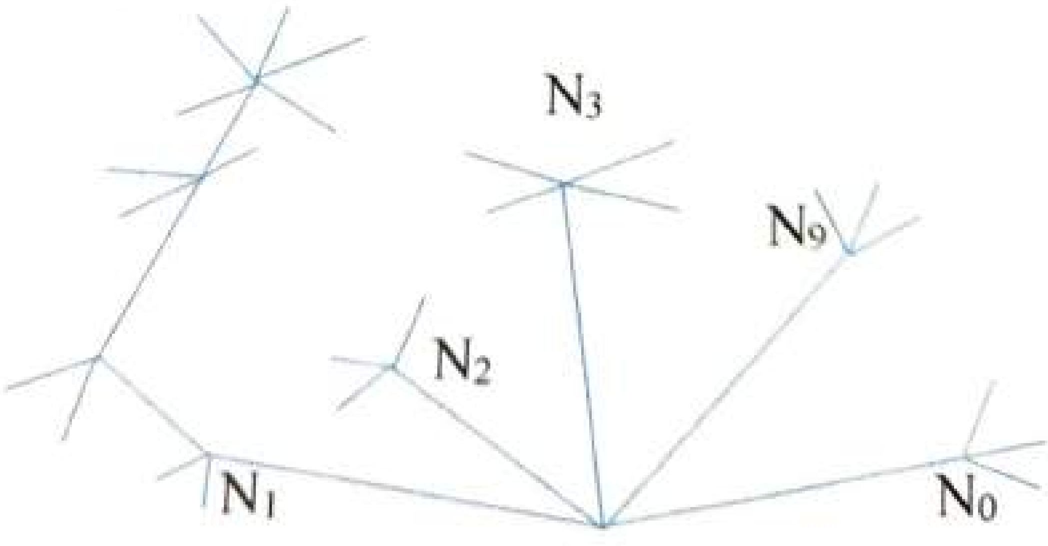


fig : 22

This tree may represent the flow of the mail. All the mails arrive at some local office at N. The most significant digit in pin code is read at N and the mail is divided into ten piles N_1, N_2,......, N_9, N_0 depending Upon the most significant digit. Each pile is further divided into ten piles according to the second most significant digit and so on. Till the mail subdivided into 10^6 possible piles, each representing a unique six digit pin code. In this tree vertex N from the rest of the vertices. Hence N is in this case, the root of the tree and the fig 22 is an example of a rooted tree.

1.2.6 PLANARITY

- **Planar Graph :**

A graph G is said to be planar if it can be drawn in a plane without intersecting the edges except its end vertices.

- Every simple planar graph contains a vertex of degree of almost five.
- Graph K_5 (complete graph in 5 vertices) and $K_{3,3}$ (a complete bipartite graph in 6 vertices) are non-planar.

- A graph is planar if and only if it contains no sub graph homeomorphic to K_5 or $K_{3,3}$.
- A graph is planar if and only if it contains no sub graph contractible to K_5 or $K_{3,3}$.

➢ **Region or Face :**

Let G be a planar graph. A region of G is the area in which any arbitrary pair of points may be joined by a curve without intersecting any edges of G.

- Let G be a connected planar graph with n vertices, m edges and r regions then n-m+r = 2. This intersection was proposed by Euler.
- Let G be a planar graph with n vertices, m edges, r regions and k components. Then n-m+r = k+1
- If G is a connected simple planar graph with $n \geq 3$ vertices and m edges then $m \leq 3n-6$.
- If in addition G has no triangle then $m \leq 2n-4$.

1.2.7 MATRIX REPRESENTATION OF GRAPH

Let G = (V, E) be an oriented graph with vertices (v_1, v_2 ,......, v_n) = V and edges (e_1, e_2,......, e_m) = E without a self-loop.

Define scalars a as follows :

a_{ij} = 1 if the edge e_i is incident with vertex v_i

= 0, otherwise

Then the matrix A = ((a_{ij})) is called Incidence matrix of G. It is denoted by A(G). Consider graph G given by

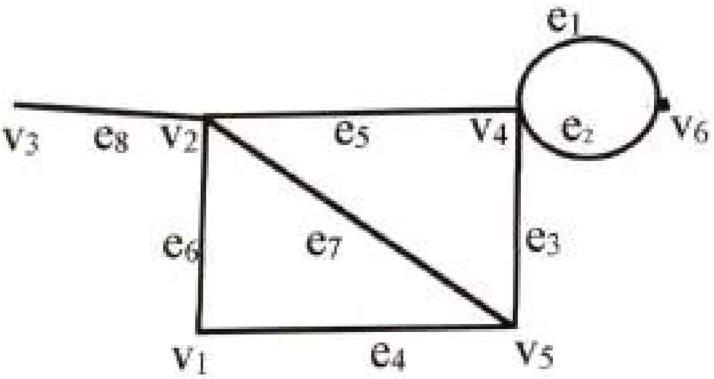


fig : 23

Here Incident matrix A(G) shall be as follows :

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
v_1	0	0	0	1	0	1	0	0
v_2	0	0	0	0	1	1	1	1
v_3	0	0	0	0	0	0	0	1
v_4	1	1	1	0	1	0	0	0
v_5	0	0	1	1	0	0	1	0
v_6	1	1	0	0	0	0	0	0

Here Incident matrix $A(G) = [\,(a_{ij})\,]\ 6 \times 8$ consists of the scalars 0 and 1 as shown above. On the other hand, if we are given a matrix of the form A(G) as above. We can construct the graph G without difficulty.

In respect of incidence matrix A(G) of a connected graph G. We have the following informations.

i) Each column of A(G) has exactly two 1's because every edge is incident with exactly two vertices.

ii) The number 1's in each row is equal to the degree of the corresponding vertex.

iii) A row with all 0's represents an isolated vertex.

iv) Parallel edges produce identical columns. In case of a disconnected graph G that consists of two components g_1 and g_2 , the incidence matrix A(G) can be written in m block Diagonal from A(G) = where $A(g_i)$ is the incidence matrix corresponding to the subgraph g_i (i = 1,2). This is the result of the fact that no edge in g_1 is incident with any vertex of g_2 and vice-versa.

$$\begin{pmatrix} A(g_1) & 0 \\ 0 & A(g_2) \end{pmatrix}$$

➢ Adjacency Matrix :

Sometimes it is more convenient to represent a graph by its adjacency matrix.

Let G be a graph of n vertices without any parallel edges. Then adjacency matrix of G, denoted by X is a symmetric binary matrix with X=(X_{ij}) n x m defined over ring of integers by the rule.

x_{ij} = 1, if there is an edge between vertices

= 0, otherwise

For example, consider a graph $G = (V, E)$ where $V = (v_1, v_2, v_3, v_4, v_5, v_6)$ and $E = (e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8)$ having no parallel edges as given by

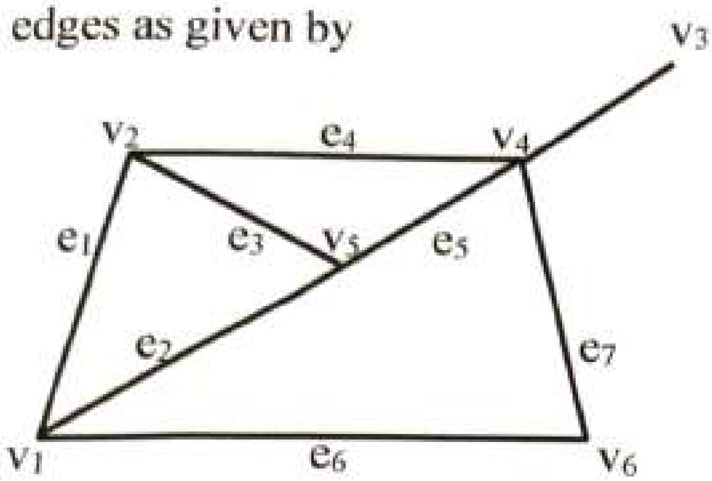


fig : 24

Then adjacency matrix X for G reads as -

$$X = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

> **Properties of adjacency matrix :**

i) The matrix is symmetric and is a square matrix ; with entries either 0 or 1.

ii) The entries along the principal diagonal of adjacency matrix X are all zero.

iii) The degree of a vertex equals to the number of 1's in the corresponding row and column in X.

iv) Permutations of rows and columns imply re-ordering of vertices. So if two rows are interchanged in X, the corresponding columns also interchanged.

v) A graph G if disconnected has two components g_1 and g_2 iff

$$X = \begin{pmatrix} X(g_1) & 0 \\ 0 & X(g_2) \end{pmatrix}$$

where $X(g_1)$ and $X(g_2)$ are adjacency matrices of the components of g_1 and g_2 respectively.

Further given any square symmetric binary matrix X of order n, we can construct a graph G of n vertices without parallel edges such that $X = X(G)$.

1.2.8 ECCENTRICITY AND CENTER

- **Definition of center :**

A vertex V with minimum E(v) in G is called a center of G

Let G be a connected graph and G = (V, E) and let v ϵ V.

- **Definition of eccentricity :**

The eccentricity E(v) of V in G is defined as E(v) = max d (v , v_i)

Thus from the definition above, it follows that E(v) denotes the distance of v to a vertex furthest away from v in G.

Consider a graph as,

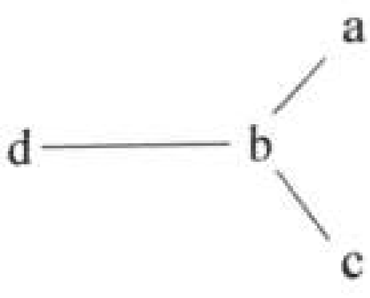


fig : 25

In this graph fig 25, b is a center because E(b) is minimum.

E(a) = 2, E(b) =1, E(c) = 2, E(d) = 2

Where as a regular polygon taken to represent a graph, shall have each vertex as a center of the graph.

SECTION 2:

APPLICATION OF GRAPH THEORY

Because of its inherent simplicity, graph theory has a very wide range of applications in Physics, Chemistry, Biological Science, Computer Science, Engineering, Social Science and other areas. Every binary structural relation can be represented by a graph. In order to model several types of relation process in physical, biological, social and information system, graph may be involved.

The graphs concern the relationship among vertices and edges. Almost any problem related to our real life involving discrete objects and a relation among them can be converted in a problem of graph theory. Perhaps, the beginning and the best known example of graph theory is **The Konigsberg's Bridge Problem** which was considered by Leonhard Euler, a great Swiss Mathematician. This problem has been discussed in the 2nd page and its negative solution is in page 16. Besides this renowned problem, there are hundreds of such applications of graph theory like Electrical Network Problem, Seating Problem, Travelling Salesman Problem, Transportation Problem and many more.

➢ 2.1 Graph Representation of Electrical Network :

In 1847, Gustav Robert Kirchhoff, a German physicist developed graph theory during the time when he was on electrical network. An Electrical Network is a collection of inter connected electrical elements (or devices) such as resistors, capacitors, inductors, diodes, transistors, vaccum tubes, switches, storage batteries, transformers, delay lines, power resources and the like.

Properties of an electrical network are functions of only two factors : The nature and value of the elements forming the network, such as resistors, inductors, transistors and so forth. The way these elements are connected together, i.e the topology of the network.

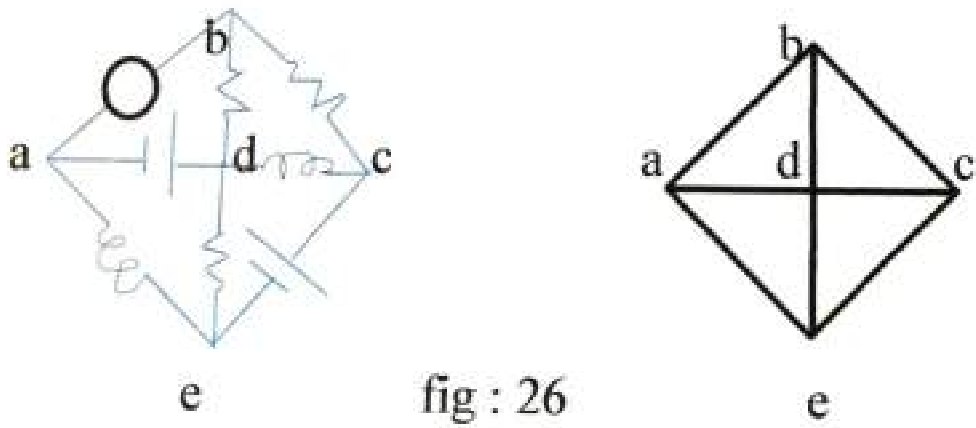
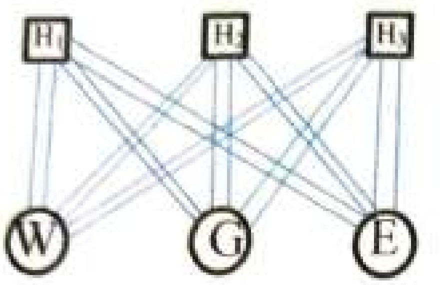


fig : 26

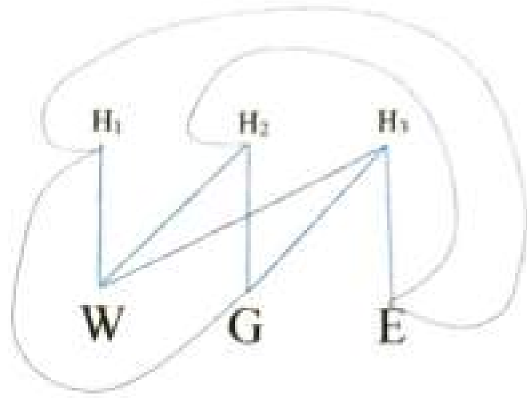
In drawing a graph of an electrical network, the junctions are represented by vertices and branches are represented by edges.

➢ 2.2 Utilities Problem :

There are three houses (fig 27) H_1 , H_2 and H_3 , each to be connected to each of the there utilities - water(W), Gas(G), and Electricity(E) - by means of conduits. Here the problem is - Is it possible to make such connections without any crossovers of the conduits?



(a)



(b)

Fig : 27 (a) The utilities problem and (b) its graph

In fig 27(b), the conduits are shown as edges while the houses and utility supply centres are vertices.This graph is known as $K_{3,3}$, Kuratowski's second graph. $K_{3,3}$, the complete bipartite graph is a simple connected graph having 6 vertices and 9 edges. Then $n_v = 6$, and $n_e = 9$. Therefore $2n_v - 4 = 12 - 4 = 8 < n_e$, i.e $n_e > 2n_v - 4$. We know that if a connected bipartite graph graph satisfies the inequality $n_e > 2n_v - 4$, it will be non-planar. Hence, it is impossible to make such connections without any crossover of the conduits.

➢ **2.3 Seating Problem :**

Nine members of a new club meet each day for lunch at a round table. They decide to sit such that every lunch member has different neighbours at each lunch. How many days can this arrangement last?

This situation can be represented by a graph with none vertices such that each vertex represents a member, and an edge joining two vertices represents the relationship of sitting next to each other.

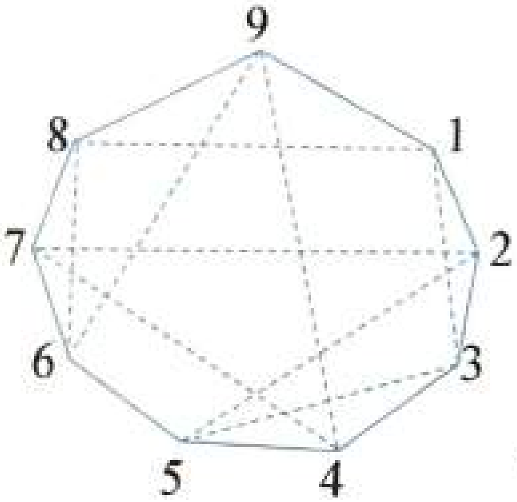


fig : 28: arrangements at a dinner table

Fig 28 shows two possible seat arrangements - these are 1234567891 (solid lines) and 1352749681 (dashed lines). It can be shown by graph - theoretic considerations that there are only two more arrangements possible.

They are 1573928461 and 1795836241. In general it can be shown that for n people the number of such possible arrangements is

(n-1)/2, if n is odd and (n-2)/2. Here, in fig 28, n = 9 (odd), then (n-1)/2 = (9-1)/2 =4.

➢ **2.4 <u>The Travelling Salesman Problem</u> :**

This problem was first considered by an American Mathematics Merrill Meeks Flood in 1930 in order to solve a school bus routing problem. But it was first formulated mathematically by William Rowan Hamilton, an Irish Mathematician, and Thomas Penyngton, Kirkman, a British Mathematician known for Kirkman's school girl problem.

➢ **The Problem is stated as follows :**

A salesman is required to visit number of cities during a trip. Given the distances between the cities, in what order should be travel so as to visit every city precisely once and return home, with the minimum mileage travelled?

Representing the cities by vertices and the roads between them by edges, we get a graph. In this graph, with every e_i there is associated a real number, (the distance in miles, say), $W(e_i)$.This graph has numerous Hamiltonian circuits. The total number of different of different Hamiltonian circuits in a complete graph of n vertices can be shown to be $(n-1)!/2$.

Theoretically, this problem can always be solved by enumerating all $(n-1)!/2$. Hamiltonian circuits, calculating the distance travelled in each, and then picking the shortest one. But, for a large value of n, the labour involved is too great. There are also available methods of solution. Such a method of solution has been used below.

- **Examples of Travelling Salesman Problem and it solution :-**

A salesman has to visit give cities namely A, B, C, D and E. Starting from the home city A and visiting each city exactly once, he has to return to the city A. The distances from one city to another are given below. Find the optimal route and minimum distance of the route.

	A	B	C	D	E
A	-	4	7	3	4
B	4	-	6	3	4
C	7	6	-	8	5
D	3	3	8	-	8
E	4	4	5	8	-

➤ **Solution :** The given travelling Salesman Problem can be represented by the following regular weighted graph.

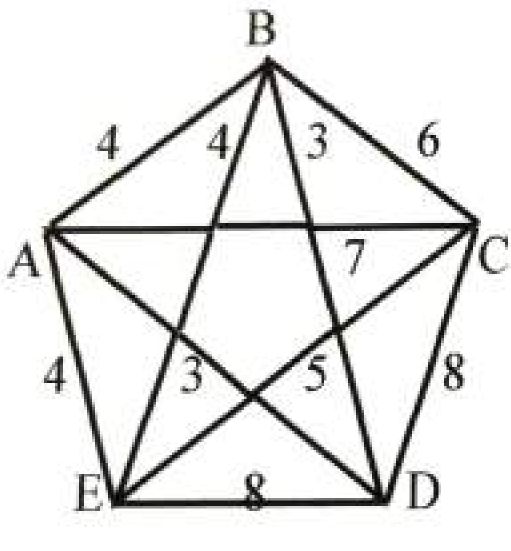


fig :29

It is a complete weighted graph with 5 vertices (considering five cities as five vertices of the graph). Since A is the home city, the salesman has to start from the city A.

Step 1 : The adjacent vertices of A are B, C, D and E whose distances from A are 4, 7, 3 and 4 units respectively. Since min (4, 7, 3, 4) = 3, D is the nearest vertex of A. So, we choose D and follow A→D.

Step 2 : The adjacent vertices of D are A, B, C, D and E with distances 3, 3, 8 and 8 units respectively from D. We cannot choose A because he has to return to A visiting all the vertices exactly once. Again, min (3, 8, 8) = 3. Thus, we have to choose B, because B is the nearest adjacent vertex of D among B, C, E. Consequently, we follow A→D→B.

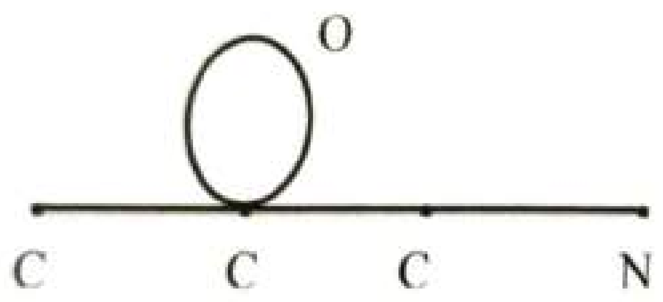
Step 3 : A and D are already visited. So, the next adjacent vertices of B are C and E and min (6,4) = 4. It follows that E is the nearest adjacent vertex. That's why we choose it and follow A→ D→ B→ E.

Step 4 : There are exactly one unvisited vertex C. So, we have to visit C and finally return to A. Hence, the optimal route with home city A is given by A→D→B→E→C→A and the corresponding minimum distance is (3+3+4+5+7) i.e 22 units.

➢ 2.5 **Graphs in Chemistry** :

Given a chemical substance and some of its properties (such as molecular weight. chemical composition, mass spectrum etc.), the chemist would like to find out if this substance is a known compound. Therefore, a standard representation which must be compact, unambiguous and amenable to classification, is essential.

A chemical compound can be represented by means of a connected graph, with the atoms as the vertices and the bonds between them as edges. For compactness the hydrogen atoms are omitted from the representation, as they are implied by every unused valence of the other atoms. For an example,



Here, the structural graph of amino acetone ($C_3H_7NO_3$) with its H atoms stripped off is shown in fig 31. [valence for C is 4, for N is 3, for O is 2].Though the structural graph of a chemical compound does not contain all the informations but it contains more than the molecular formula does.

For example.

the molecular formula $C_{10}H_{22}$ can denote any of its 75 isomers (75 being the number of unlabeled trees with 10 vertices and with no vertex of degree five or more), while the graph specifies the exact isomer.

- **Matching of Chemical Structure** :

The problem of determining whether or not two chemical formula having the same chemical formula, are identical is the same as the graph isomorphism problem, considerably simplified by the labels of the vertices.

The underlying idea behind this algorithm is to use various properties (such as labels, degrees, adjacencies etc.) of vertices in the two graphs to generate pairs of vertex subsets, which must match if the graphs are to be isomorphic.

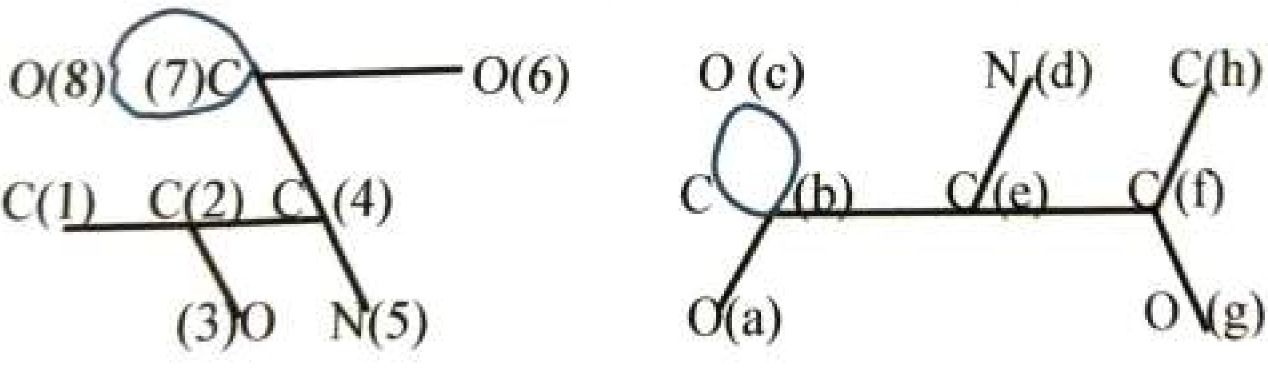


fig : 31

Let us determine whether or not the two molecules in fig 32 are identical (H atoms are not shown as fig 31).

The vertices are arbitrarily named (1), (2),......, (8) in G and (a),(b),......, (h) in J. From the graphs shown in fig 32, we can conclude that G and J are isomorphic by its properties.

The vertex correspondence is :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ h & f & g & e & d & a & b & c \end{pmatrix}$$

2.6 Graph in sociological Structures :

Digraphs under the name sociograms have been used to represent relationship among individuals in a society. Members are represented by vertices and the relationship by directed edges. Connectedness, separability, complete sub digraphs , size of fragments, and so forth in a sociogram can be given immediate significance.

Graph theory has also been used in economics, logistics, cybernetics, artificial intelligence, pattern recognition, genetics, reliability theory, fault diagnosis in computers, studying of computer memory, and the study of Martian Canals. A mathematical model of disarmament has been attempted with graph theory and so have the conflict in the Middle East and the structure if Mozart's Opera, cosi fan tutte and thus goes the ever increasing List of applications of graph theory. Admittedly, in some of these applications, a graph is used only as a means of visual representation, and no more than a trivial use is made of graph theory itself.

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CONCLUSION

In this project, we have presented applications of graph theory in various fields. First, we have discussed its preliminaries to understand its applications more clearly. There are hundreds of applications of graph theory. Graph theory is applied to solve any practical problem like electrical network analysis, circuit layout, data structures, operations research, social science. An engineer often finds that those real life problems that can be modeled into graphs small enough to be worked on by hand are so also small enough to be solved by means other than graph theory. In fact, the high-speed digital computer is one of the reasons for the growth of interest in graph theory. Graph theory is also applied in google maps. The last eight decades have witnessed an upsurge of interest and activity in graph theory, particularly among applied mathematicians and engineers. We have shown here a few examples of problems related to graph theory. The interest in graph theory is increasing day by day. Nowadays, even in games, which we play in smart phones, graph theory has been used to solve these puzzles.

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Sl. N	Name of the st	Roll No.	Title of Project	Marks in Presentation (out of 20)	Marks in Viva Voce (out of 15)
1	Ankita Sen	200340200001	Applications of graph theory	15	15

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